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1987 J. Phys. A: Math. Gen. 20 4717

(http://iopscience.iop.org/0305-4470/20/14/016)

J. Phys. A: Math. Gen. 20 (1987) 4717-4732. Printed in the UK

# A causal account of non-local Einstein–Podolsky–Rosen spin correlations

C Dewdney†, P R Holland‡ and A Kyprianidis‡

† Department of Applied Physics, Portsmouth Polytechnic, Park Building, Portsmouth PO1 2DZ, UK

‡ Laboratoire de Physique Théorique, Institut Henri Poincaré, 11 rue P et M Curie, 75231 Paris Cedex 05, France

Received 20 January 1987

**Abstract.** We give a causal interpretation of a double Stern-Gerlach experiment on the basis of spacetime solutions to the Pauli equation. For an initial singlet state, we determine the continuous particle trajectories and spin vector orientations. The graphical results exemplify how non-local actions of the quantum potential and quantum torque give rise to a correlated evolution of dynamical variables.

## 1. The causal description of the Pauli spinor

In a recent paper [1] we illustrated a model proposed by Bohm *et al* [2] and also by Takabayasi [3], in which the Pauli equation can be interpreted as describing the motion of a spinning particle in the sense that well defined individual particle trajectories can be calculated along with actual spin vector orientations. According to this approach a particle in a 'spin up' state with respect to some $z$ axis has a spin vector which points along this $z$ direction; the $x$ and $y$ components of the spin vector are not undetermined but actually zero. However, if the spin component is measured using a Stern-Gerlach apparatus oriented, for example, along the $x$ direction the result 'up $x$' and 'down $x$' will be found with equal frequency in this state. A calculation shows that in this case the beam bifurcates along a central plane perpendicular to the analysing direction as two separating 'up' and 'down' beams are formed. The outcome in a particular case is determined by the uncontrollable actual position of the particle relative to this bifurcation plane at the entrance slit to the field. The particle enters one beam or the other as a result of the action of a spin-dependent 'quantum force' and as the beams separate a 'quantum torque' rotates the spin vector to lie either along or opposed to the direction of the analysing field. In this way the quantum phenomena associated with spin can be understood in a manner closer, in some ways, to our customary forms of description than is usually the case, which highlights the essential differences between quantum and classical phenomena. Such a description is possible since the particle and the spinor wave are assumed to have equal ontological status. The quantum force and quantum torque which act on the particle coordinates in this model arise when the Pauli equation is recast in pseudoclassical form. Using this approach we were able to give a consistent description of the process of the measurement of the spin as outlined above, and of spin superposition in neutron interferometry [4].

0305-4470/87/144717+16$02.50  

In the approach described above the orientation of the spin vector is defined in terms of the Euler angles $\theta$, $\phi$ and $\chi$, by writing the Pauli two-component spinor as

$$\psi = R\{\cos\tfrac{1}{2}\theta \exp[\mathrm{i}(\phi+\chi)/2]u_+ + \mathrm{i}\sin\tfrac{1}{2}\theta\exp[-\mathrm{i}(\phi-\chi)/2]u_-\} \tag{1.1}$$

where $R$ is a spatial amplitude and $u_+$ and $u_-$ are the 'spin up' and 'spin down' eigenfunctions

$$u_+\begin{pmatrix}1\\0\end{pmatrix} \qquad u_-\begin{pmatrix}0\\1\end{pmatrix}.$$

The probability density is

$$\rho = \psi^+\psi = R^2$$

and the current

$$j = \frac{\hbar}{2m\mathrm{i}}(\psi^+\nabla\psi - (\nabla\psi^+)\psi) - \frac{e}{mc}A\rho$$

yields the velocity

$$v = \frac{\hbar}{2m}(\nabla\chi + \cos\theta\nabla\phi) - \frac{e}{mc}A$$

from which trajectories may be calculated by solving $v = \dot{x}(t)$. The spin vector is defined to be

$$s = \tfrac{1}{2}\hbar(\psi^+\sigma\psi/\rho) = \tfrac{1}{2}\hbar(\sin\theta\sin\phi, \sin\theta\cos\phi, \cos\theta). \tag{1.2}$$

In this paper we extend this causal approach to spin-$\frac{1}{2}$ systems to embrace a treatment of Bohm's [5] version of the Einstein-Podolsky-Rosen (EPR) experiment [6] in which spin measurements are carried out on each spin-$\frac{1}{2}$ particle forming a singlet state. Bohm's proposal has been of great historical importance in the debate on hidden variables and non-locality [7], but hitherto the precise nature of the physical process that lies behind the 'non-local' correlations in the spins of the particles has remained unclear. With the aim of clarifying the situation, and in particular refuting some of the arbitrary assertions which are made concerning this process, we present here plots of the correlated motions of the particles and the evolution of their spin vectors. Our conclusion is that whilst the model discussed here is an idealised one, it provides an insight into the meaning of 'non-locality' in a way that no other interpretation of quantum mechanics has managed to do.

## 2. Causal approach to the two-body problem with spin

Consider a system of two spin-$\frac{1}{2}$ particles of masses $m_1$, $m_2$ and charges $e_1$, $e_2$, respectively, which are placed in external electromagnetic fields and possibly interact. The two-body Pauli equation is

$$\mathrm{i}\hbar\frac{\partial\psi}{\partial t} = \left[-\frac{\hbar^2}{2m_1}\left(\nabla_1 - \frac{\mathrm{i}e}{\hbar c}A_1(x_1, x_2)\right)^2 - \frac{\hbar^2}{2m_2}\left(\nabla_2 - \frac{\mathrm{i}e}{\hbar c}A_2(x_1, x_2)\right)^2 + W_1 + W_2 + V\right]\psi \tag{2.1}$$

where $x_1, x_2$ are the coordinates of particles 1 and 2, $(\psi) = \psi_{ab}(x_1, x_2, t)$ is the wavefunction of the system (a representation of $SU(2)\otimes SU(2)$), $V = V(x_1, x_2, t)$ is the total external plus interaction scalar potential,

$$W_1 = W_1(\boldsymbol{x}_1, \boldsymbol{x}_2, t) = \mu_1 \boldsymbol{H}_1(\boldsymbol{x}_1, \boldsymbol{x}_2) \cdot \boldsymbol{\sigma}_1$$
$$W_2 = W_2(\boldsymbol{x}_1, \boldsymbol{x}_2, t) = \mu_2 \boldsymbol{H}_2(\boldsymbol{x}_1, \boldsymbol{x}_2) \cdot \boldsymbol{\sigma}_2 \tag{2.2}$$

where $\mu_1, \mu_2$ are the magnetic moments of the particles with $\boldsymbol{H}_1 = \nabla_1 \times \boldsymbol{A}_1$, $\boldsymbol{H}_2 = \nabla_2 \times \boldsymbol{A}_2$, and $\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2$ are two sets of Pauli matrices which commute and operate independently on the spin indices $a$, $b$, respectively.

Writing

$$\psi_{ab} = R\, \mathrm{e}^{\mathrm{i}S/\hbar} \phi_{ab} \tag{2.3}$$

where $R$ and $S$ are real amplitude and phase functions, respectively, and $\phi^\dagger\phi = 1$, we may deduce from (2.1) by contracting with $\psi^\dagger$ and separating into real and imaginary parts a generalised Hamilton-Jacobi equation

$$\frac{\partial S}{\partial t} - \mathrm{i}\hbar\phi^\dagger \frac{\partial \phi}{\partial t} + \tfrac{1}{2} m_1 \boldsymbol{v}_1^2 + \tfrac{1}{2} m_2 \boldsymbol{v}_2^2 + Q_1 + Q_2 + H_{1s} + H_{2s}$$
$$+ (2/\hbar)\mu_1 \boldsymbol{H}_1 \cdot \boldsymbol{s}_1 + (2/\hbar)\mu_2 \boldsymbol{H}_2 \cdot \boldsymbol{s}_2 + V = 0 \tag{2.4}$$

and a continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla_1 \cdot (\rho \boldsymbol{v}_1) + \nabla_2 \cdot (\rho \boldsymbol{v}_2) = 0. \tag{2.5}$$

Here $\rho = \psi^\dagger\psi = R^2$ is the configuration space probability density,

$$\boldsymbol{v}_i = \frac{-\mathrm{i}\hbar}{2m_i\rho} \psi^\dagger \overleftrightarrow{\nabla}_i \psi - \frac{e_i}{m_i c} \boldsymbol{A}_i = \frac{1}{m_i}\left(\nabla_i S - \mathrm{i}\hbar\phi^\dagger \nabla_i \phi - \frac{e_i}{c} \boldsymbol{A}_i\right) \qquad i = 1, 2 \tag{2.6}$$

are the velocities of the particles which contain spin-dependent contributions, $Q_i = -(\hbar^2/2m_i)\nabla_i^2 R/R$ $(i = 1, 2)$ are the usual quantum potentials which arise in the spinless two-body problem,

$$H_{is} = \frac{\hbar^2}{2m_i}[\nabla_i \phi^\dagger \cdot \nabla_i \phi + (\phi^\dagger \nabla_i \phi)^2] \qquad i = 1, 2 \tag{2.7}$$

are spin-dependent additions to the quantum potentials, and

$$\boldsymbol{s}_i = \tfrac{1}{2}\hbar\phi^\dagger \boldsymbol{\sigma}_i \phi = \tfrac{1}{2}\hbar\psi^\dagger \boldsymbol{\sigma}_i \psi/\rho \qquad i = 1, 2 \tag{2.8}$$

are the vectors which we shall adopt as describing the local spin orientation of each particle. The total energy of the system

$$\frac{\partial S}{\partial t} - \mathrm{i}\hbar\phi^\dagger \frac{\partial \phi}{\partial t} \tag{2.9}$$

is clearly spin dependent, and these equations generally imply a spin-orbit coupling. Notice that all the above functions are in general dependent on the coordinates of both particles. Thus the trajectories of the particles, defined by the solutions to

$$\mathrm{d}\boldsymbol{x}_i/\mathrm{d}t = \boldsymbol{v}_i \qquad i = 1, 2 \tag{2.10}$$

(given the initial positions) depend irreducibly on each other and on the total quantum state. It is easy to to see that the trajectories and spin vectors of the two particles will only evolve independently when the wavefunction factories:

$$\psi_{ab}(\boldsymbol{x}_1, \boldsymbol{x}_2, t) = \psi_{1a}(\boldsymbol{x}_1, t)\psi_{2b}(\boldsymbol{x}_2, t) \tag{2.11}$$

where $\psi_{ia} = R_i e^{iS_i/\hbar}\phi_{ia}$, $\phi_i^\dagger\phi_i = 1$, $i = 1, 2$, is given by (1.1) for each $i$. In this case the quantum potentials $Q_i + H_{is}$ decompose into a sum of functions, each associated with just one of the particles and the spin vectors take the form (1.2) for each $i$. When (2.11) does not hold the spin-dependent quantum potentials act to bring about correlations in the motions since they give rise to quantum forces and torques which are functions of both coordinates. This may be seen explicitly from the equations of precession of the spin vectors:

$$\mathrm{d}\boldsymbol{s}_i/\mathrm{d}t = \boldsymbol{T}_i + (2\mu_i/\hbar)\boldsymbol{H}_i \times \boldsymbol{s}_i \qquad i = 1, 2 \tag{2.12}$$

where $\boldsymbol{T}_i$, $i = 1, 2$, are quantum torques:

$$\begin{aligned} T_{1k} = {} & \frac{1}{2\rho m_1}\varepsilon_{ijk}\{s_{1i}\partial_{1l}(\rho\partial_{1l}s_{1j}) + X_{ir}\partial_{1l}(\rho\partial_{1l}X_{jr})\} \\ & + \frac{1}{2\rho m_2}\varepsilon_{ijk}\{s_{1i}\partial_{2l}(\rho\partial_{2l}s_{1j}) + X_{ir}\partial_{2l}(\rho\partial_{2l}X_{jr})\} \end{aligned} \tag{2.13}$$

$$T_{2k} = [1 \leftrightarrow 2]$$

with $X_{ij} = (\hbar/2\rho)\psi^\dagger\sigma_{1i}\sigma_{2j}\psi$, $\mathrm{d}/\mathrm{d}t = \partial/\partial t + \boldsymbol{v}_1\cdot\nabla_1 + \boldsymbol{v}_2\cdot\nabla_2$, and the equations of translational motion of each particle:

$$\begin{aligned} m_1\frac{\mathrm{d}v_{1j}}{\mathrm{d}t} = {} & -\partial_{1j}\left(Q_1 + Q_2 + H_{1s} + H_{2s} + V + \frac{2\mu_1}{\hbar}\boldsymbol{H}_1\cdot\boldsymbol{s}_1 + \frac{2\mu_2}{\hbar}\boldsymbol{H}_2\cdot\boldsymbol{s}_2\right) \\ & + F_{1j} + v_{2k}\left(\frac{e_2}{c}\partial_{1j}A_{2k} - \frac{e_1}{c}\partial_{2k}A_{1j}\right) \\ & + \mathrm{i}\hbar\left(\partial_{1j}\phi^\dagger\frac{\mathrm{d}\phi^\dagger}{\mathrm{d}t} - \frac{\mathrm{d}\phi^\dagger}{\mathrm{d}t}\partial_{1j}\phi\right) \end{aligned} \tag{2.14}$$

$$m_2\frac{\mathrm{d}v_{2j}}{\mathrm{d}t} = [1 \leftrightarrow 2]$$

where

$$F_{1j} = -\frac{e_1}{c}\frac{\partial A_{1j}}{\partial t} + \frac{e_1}{c}v_{1k}(\partial_{1j}A_{1k} - \partial_{1k}A_{1j})$$

and $\mathrm{d}\phi/\mathrm{d}t$ is given by (2.12).

It is important to emphasise that in the approach proposed here it is possible to distinguish the particles in a many-body system by their individual trajectories (including when the particles are identical in the quantum mechanical sense). This means that in a singlet state, for which it is usually stated that before measurement the total angular momentum is well defined whereas the individual spins are indefinite and that after measurement each individual spin is definite but the total spin is indefinite, each of the particles does in fact possess a definite (but continuously variable) spin vector at all times, both before, during and after measurement. This of course is because the orthodox account of the EPR experiment only makes statements concerning the eigenvalues of operators, with well defined spins only coming into existence as a result of 'measurement'. As we have seen [1], the effect of the measurement is simply to continuously transform the value of a quantity which already existed.

## 3. Application to the EPR problem

The basic set-up is shown in figure 1. A pair of spin-$\frac{1}{2}$ particles of mass $m$ and magnetic moment $\mu$ are formed at O in a simultaneous eigenstate of the spin operator in the $z$ direction $(\hbar/2)(\sigma_{z_1}+\sigma_{z_2})$ and the total spin operator $(\hbar^2/4)(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)^2$, with eigenvalue zero. The particles separate in the $y$ direction and pass through Gaussian slits oriented so as to produce packets in the directions of the analysing fields of two identical Stern–Gerlach devices. Magnet 2 is set to measure the spin in the $z$ direction, and magnet 1 has been rotated anticlockwise through an angle $\delta$ about the $y$ axis so that it has a gradient in the $z'$ direction.

At the entrance to the magnets the wavefunction is†

$$\psi_0=f_1(z_1')f_2(z_2)\frac{1}{\sqrt{2}}(u_+v_- - u_-v_+) \tag{3.1}$$

where $f_1(z_1')$, $f_2(z_2)$ are normalised parckets, $z_1'$ and $z_2$ are the coordinates of particles 1 and 2 in the $z'$ and $z$ directions, respectively, and $\sigma_{z_1}u_\pm=\pm u_\pm$, $\sigma_{z_2}v_\pm=\pm v_\pm$. The state (3.1) predicts the following well known expectation value for the correlations of spins measured in the $z$, $z'$ directions:

$$\langle\sigma_{z_1'}\sigma_{z_2}\rangle=-\cos\delta \tag{3.2}$$

and the probabilities of the possible outcomes are given by

$$P(++)=P(--)=\tfrac{1}{2}\sin^2\delta/2,\ P(+-)=P(-+)=\tfrac{1}{2}\cos^2\delta/2. \tag{3.2a}$$

In (3.1) we have suppressed the motion in the $y$ direction since this is not relevant to the measurement process. We only assume that the particles are sufficiently far apart on the $y$ axis so that they do not interact in the usual sense, and so that the measuring devices cannot influence one another.

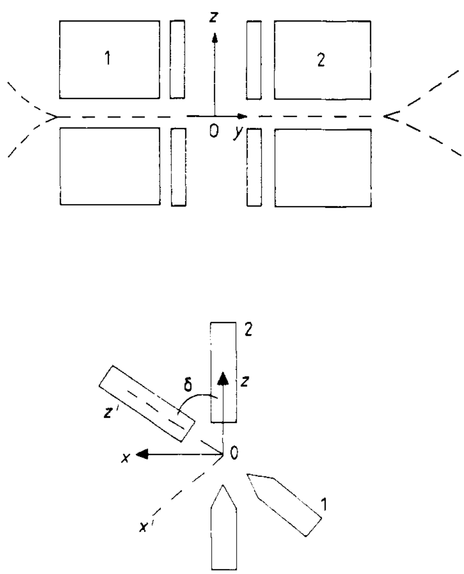


**Figure 1.** The arrangement for the Einstein–Podolsky–Rosen experiment.

† Note that we do not antisymmetrise on the coordinates so that the particles are distinguishable in the usual sense.

Equation (3.1) describes a state in which the spin is independent of position and the purpose of the Stern–Gerlach devices is to introduce couplings between the spins (the measured variables) and the positions (the apparatus coordinates). The effect of the impulsive action of the inhomogeneous fields (assumed to act simultaneously) may be determined by solving (2.1) in the approximation where the kinetic energy terms are neglected. At the exit to each magnet two superposed packets have been formed which separate in time according to motion described by the free Pauli equation along the directions of the analysing fields. The authors have not found this calculation in the literature and the details of our simplified treatment are sketched in the appendix.

Expanding in terms of products of eigenfunctions of the spin operators being measured, the normalised wavefunction at time $t$ after the interaction with the fields is

$$\begin{aligned}\psi(z_1', z_2, t) &= (\sigma\sqrt{2\pi} s_t)^{-1}\\ &\times \exp\left\{-\left[\frac{z_1'^2+z_2^2}{2}+\left(\frac{\hbar t\Delta'}{m}\right)^2\right](\varepsilon\sigma^2)^{-1}\right\}\exp\left\{\frac{\mathrm{i}\hbar t}{m\varepsilon}\left(\frac{z_1'^2+z_2^2}{2}-\frac{\Delta'^2}{\sigma^4}\right)\right\}\\ &\times(\sin\tfrac{1}{2}\delta\exp[-\hbar t\Delta'(z_1'+z_2)/m\varepsilon\sigma^2]\exp\{-\mathrm{i}[2\Delta+(z_1'+z_2)\Delta'/\varepsilon\sigma^4]\}u_+'v_+\\ &+\cos\tfrac{1}{2}\delta\exp[-\hbar t\Delta'(z_1'-z_2)/m\varepsilon\sigma^2]\exp[-\mathrm{i}(z_1'-z_2)\Delta'/\varepsilon\sigma^4]u_+'v_-\\ &-\cos\tfrac{1}{2}\delta\exp[\hbar t\Delta'(z_1'-z_2)/m\varepsilon\sigma^2]\exp[\mathrm{i}(z_1'-z_2)\Delta'/\varepsilon\sigma^4]u_-'v_+\\ &+\sin\tfrac{1}{2}\delta\exp[\hbar t\Delta'(z_1'+z_2)/m\varepsilon\sigma^2]\exp\{\mathrm{i}[2\Delta+(z_1'+z_2)\Delta'/\varepsilon\sigma^4]\}u_-'v_-)\end{aligned}\tag{3.3}$$

where $s_t=(1/\sigma^2+\mathrm{i}\hbar t/m)$, $\sigma=$ constant, $\varepsilon=1/\sigma^4+(\hbar t/m)^2$, $\Delta=\mu H_0 T/\hbar$, $\Delta'=\mu H_0' T/\hbar$, $H_0$ is the homogeneous part of the field, $H_0'$ is the field gradient and $T$ is the period of interaction.

The velocities and spin vectors at $t=0$ may be calculated from (2.6), (2.8) and (3.1) or (A2) to $\boldsymbol{v}_1=\boldsymbol{v}_2=\boldsymbol{s}_1=\boldsymbol{s}_2=0$. It may appear strange that the particles have zero true values of internal angular momentum but it should be borne in mind that in quantum mechanics the properties of individuals depend on the state of the whole of which they are a part. Each individual spin vector will possess properties different to those of the spin vector associated with a single body (which by (1.2) can never be zero). Notice in particular that the spins of the particles in each separating packet are not determined by either of the addends in the state (3.1), i.e. they are not in an initial state in which the spin of one is up (down) and the other is down (up), as one would expect in the analogous classical case.

Immediately after the interaction with the magnets, however, the particles acquire non-zero $z'$ and $z$ components of their velocities and the quantum torques (2.13) act to start the particles spinning. At time $t$ we have from (3.3) the density

$$\rho=(2\pi\sigma^2\varepsilon)^{-1}\exp\{-[z_1'^2+z_2^2+2(\hbar t\Delta'/m)^2]/\varepsilon\sigma^2\}\Omega \tag{3.4}$$

where

$$\begin{aligned}\Omega &= \sin^2\tfrac{1}{2}\delta\{\exp[2\hbar t\Delta'(z_1'+z_2)/m\varepsilon\sigma^2]+\exp[-2\hbar t\Delta'(z_1'+z_2)/m\varepsilon\sigma^2]\}\\ &+\cos^2\tfrac{1}{2}\delta\{\exp[2\hbar t\Delta'(z_1'-z_2)/m\varepsilon\sigma^2]+\exp[-2\hbar t\Delta'(z_1'-z_2)/m\varepsilon\sigma^2]\}\end{aligned}\tag{3.5}$$

and for particle 1

$$s_{x_1'}=\frac{\hbar}{2\Omega}\sin\delta\cos\left[2\left(\Delta+\frac{z_1'\Delta'}{\varepsilon\sigma^4}\right)\right][\exp(2\hbar t\Delta' z_2/m\varepsilon\sigma^2)-\exp(-2\hbar t\Delta' z_2/m\varepsilon\sigma^2)]$$

$$s_{y_1'}=\frac{\hbar}{2\Omega}\sin\delta\sin\left[2\left(\Delta+\frac{z_1'\Delta'}{\varepsilon\sigma^4}\right)\right][\exp(2\hbar t\Delta' z_2/m\varepsilon\sigma^2)-\exp(-2\hbar t\Delta' z_2/m\varepsilon\sigma^2)]$$

$$s_{z_1'}=\frac{\hbar}{2\Omega}\Big(\sin^2\tfrac{1}{2}\delta\{\exp[-2\hbar t\Delta'(z_1'+z_2)/m\varepsilon\sigma^2]-\exp[2\hbar t\Delta'(z_1'+z_2)/m\varepsilon\sigma^2]\}$$
$$+\cos^2\tfrac{1}{2}\delta\{\exp[-2\hbar t\Delta'(z_1'-z_2)/m\varepsilon\sigma^2]-\exp[2\hbar t\Delta'(z_1'-z_2)/m\varepsilon\sigma^2]\}\Big) \tag{3.6}$$

$$v_{z_1'}=\frac{z_1'\hbar^2 t}{m^2\varepsilon}-\frac{2\Delta'}{m\varepsilon\sigma^4}s_{z_1'} \tag{3.7}$$

while for particle 2

$$s_{x_2}=\frac{\hbar}{2\Omega}\sin\delta\cos\left[2\left(\Delta+\frac{z_2\Delta'}{\varepsilon\sigma^4}\right)\right][\exp(-2\hbar t\Delta' z_1'/m\varepsilon\sigma^2)-\exp(2\hbar t\Delta' z_1'/m\varepsilon\sigma^2)]$$

$$s_{y_2}=\frac{\hbar}{2\Omega}\sin\delta\sin\left[2\left(\Delta+\frac{z_2\Delta'}{\varepsilon\sigma^4}\right)\right][\exp(-2\hbar t\Delta' z_1'/m\varepsilon\sigma^2)-\exp(2\hbar t\Delta' z_1'/m\varepsilon\sigma^2)]$$

$$s_{z_2}=\frac{\hbar}{2\Omega}\Big(\sin^2\tfrac{1}{2}\delta\{\exp[-2\hbar t\Delta'(z_1'+z_2)/m\varepsilon\sigma^2]-\exp[2\hbar t\Delta'(z_1'+z_2)/m\varepsilon\sigma^2]\}$$
$$+\cos^2\tfrac{1}{2}\delta\{\exp[2\hbar t\Delta'(z_1'-z_2)/m\pi\sigma^2]-\exp[-2\hbar t\Delta'(z_1'-z_2)/m\varepsilon\sigma^4]\}\Big) \tag{3.8}$$

$$v_{z_2}=\frac{z_2\hbar^2 t}{m^2\varepsilon}-\frac{2\Delta'}{m\varepsilon\sigma^4}s_{z_2} \tag{3.9}$$

with

$$s_1^2=s_2^2=\frac{\hbar^2}{4\Omega^2}(\Omega^2-4). \tag{3.10}$$

The components of $s_1$ in the $xyz$ system are easily found from

$$s_{x_1}=s_{x_1'}\cos\delta-s_{z_1'}\sin\delta \qquad s_{y_1}=s_{y_1'} \qquad s_{z_1}=s_{x_1'}\sin\delta+s_{z_1'}\cos\delta. \tag{3.11}$$

There are of course no additions to the initial components of the velocities in the $y$ direction (which we have ignored) from the impulsive action of the fields.

The general implications of these results are as follows. As we have said, each particle undergoing measurement enters one of the separating packets at the exit to the magnet and the quantum torque acts to start the particle rotating. The spin vectors do not in general lie along the direction of the magnetic fields but eventually they do so, tending towards the values $\pm\hbar/2$ (0, 0, 1). Which result is obtained depends on the positions of both particles in the initial packets. On average the correlation comes out to be (3.2) and the probabilities (3.2$a$) are reflected by the relative proportions of trajectories which go up or down on leaving the magnets. The velocities clearly depend on the spins, as seen in (3.7) and (3.9).

To gain further insight into this non-local action, let us consider two important special cases for which we explicitly plot the particle trajectories and spin vector orientations.

### 3.1. *Measurement on particle 1 only*

The wavefunction in the case where no measurement is performed on particle 2 is

$$\psi(z_1', z_2, t) = (\sqrt{2\pi}\sigma s_t)^{-1} \exp\left\{-\left[z_1'^2 + z_2^2 + \left(\frac{\hbar t\Delta'}{m}\right)^2\right](2\varepsilon\sigma^2)^{-1}\right\}$$
$$\times \exp\left[\frac{i\hbar t}{2m\varepsilon}\left(z_1'^2 + z_2^2 - \frac{\Delta'^2}{\sigma^4}\right)\right]$$
$$\times\left\{\exp(-\hbar t\Delta' z_1'/m\varepsilon\sigma^2)\exp\left[-i\left(\Delta + \frac{z_1'\Delta'}{\varepsilon\sigma^4}\right)\right]u_+'v_-'\right.$$
$$\left. - \exp(\hbar t\Delta' z_1'/m\varepsilon\sigma^2)\exp\left[i\left(\Delta + \frac{z_1'\Delta'}{\varepsilon\sigma^4}\right)\right]u_-'v_+'\right\} \tag{3.12}$$

where $v_+' = \cos\frac{1}{2}\delta v_+ - \sin\frac{1}{2}\delta v_-$, $v_-' = \sin\frac{1}{2}\delta v_+ + \cos\frac{1}{2}\delta v_-$, which implies that

$$s_{x_1'} = s_{y_1'} = 0 \qquad s_{z_1'} = \frac{\hbar}{2\Omega'}[\exp(-2\hbar t\Delta' z_1'/m\varepsilon\sigma^2) - \exp(2\hbar t\Delta' z_1'/m\varepsilon\sigma^2)] \tag{3.13}$$

$$v_{z_1'} = \frac{z_1'\hbar^2 t}{m^2\varepsilon} - \frac{2\Delta'}{m\varepsilon\sigma^4}s_{z_1'} \tag{3.14}$$

$$s_{x_2} = \sin\delta s_{z_1'} \qquad s_{x_2'} = 0 \qquad s_{y_2} = s_{y_2'} = 0 \qquad s_{z_2} = -\cos\delta s_{z_1'} \qquad s_{z_2'} = -s_{z_1'} \tag{3.15}$$

$$v_{z_2} = z_2\frac{\hbar^2 t}{m^2\varepsilon} \tag{3.16}$$

where $\Omega' = [\exp(2\hbar t\Delta' z_1'/m\varepsilon\sigma^2) + \exp(-2\hbar t\Delta' z_1'/m\varepsilon\sigma^2)]$.

We see from (3.13) and (3.14) that the behaviour of particle 1 is independent of particle 2; the velocity and spins depend only on $z_1'$ and they take a form similar to the expressions obtained for a single free particle passed through a Stern-Gerlach device [1], whose initial spin vector lies in the $x'y'$ plane (although in the one-body case the spin is never zero).

In particular the spin vector points along the direction of the field and changes continuously from 0 to $\hbar/2(-\hbar/2)$ if the initial position on the $z'$ axis is above (below) $z'=0$. The fate of particle 2 on the other hand is dependent on the motion of 1. From (3.16) it is seen that the *trajectory* of 2 is unaffected by 1, and we may solve to find

$$z_2(t) = z_2(0)\sigma^2\left[\frac{1}{\sigma^4} + \left(\frac{\hbar t}{m}\right)^2\right]^{1/2}$$

i.e. a hyperbola which follows from the natural spread of the wavepacket. The component of the *spin vector* of 2 which lies along the $z'$ direction however changes continuously from zero to a finite value ($\mp\hbar/2$) simultaneously with the commencement of rotation of 1, and is at all times equal and opposite to the spin of particle 1. The spin of 2 thus depends sensitively on the position of particle 1 through the spin vector of 1, as can be seen from (3.15), whereas the respective trajectories depend solely on the initial positions of the particles and the local environment.

When the beam containing particle 1 splits at the exit to the magnet and the particle enters one or other of the separating packets, the beam containing particle 2 does not split—particle 2 remains in the same packet, but now it is rotating about the $z$ axis due to the non-local action of the quantum torque (2.13).

The sense of rotation is determined by the initial position of particle 1. If particle 1 is in the upper (lower) half of the bifurcating beam, it will rotate clockwise (anticlockwise) about the $z'$ axis and will be in a spin up (down) eigenstate of $\sigma_{z'}$. Particle 2 will have the opposite sense of rotation regardless of its position. If we subsequently pass particle 2 through a Stern-Gerlach device oriented in the $z'$ direction, then we will of course obtain the opposite result to that found for particle 1, since all trajectories in a spin up (down) eigenstate join the upper (lower) beam [1].

The trajectories and spin vector magnitudes (indicated by the length of the arrow which always lies in the $z'$ direction) are shown in figure 2.

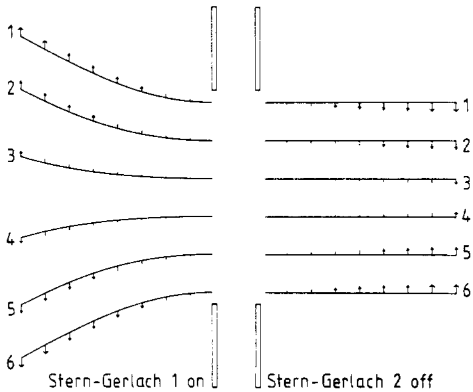


**Figure 2.** Trajectories and correlated spin vector orientations for two particles initially in a singlet state after the impulsive measurement of the $z'$ component of the spin of particle 1 only.

### *3.2. Magnets aligned ($\delta = 0$)*

In this case only the $u_+ v_-$, $u_- v_+$ terms survive in (3.3) so that $s_{x_1} = s_{y_1} = s_{x_2} = s_{y_2} = 0$,

$$s_{z_1} = -s_{z_2} = \frac{\hbar}{2\Omega}\{\exp[-2\hbar t\Delta'(z_1 - z_2)/m\varepsilon\sigma^2] - \exp[2\hbar t\Delta'(z_1 - z_2)/m\varepsilon\sigma^2]\} \tag{3.17}$$

and

$$v_{z_1} = \frac{z_1 \hbar^2 t}{m^2 \varepsilon} - \frac{2\Delta'}{m\varepsilon\sigma^4} s_{z_1} \qquad v_{z_2} = z_2 \frac{\hbar^2 t}{m^2 \varepsilon} - \frac{2\Delta'}{m\varepsilon\sigma^4} s_{z_2}. \tag{3.18}$$

The motion of each particle for any pair of trajectories depends sensitively on the choice of both initial positions at the entrance slits to the Stern-Gerlach devices. Equation (3.17) shows, however, that the spins always come out to be opposite regardless of the initial positions. The results plotted in figure 3 were calculated by taking the initial position of particle 1 to be fixed in each case and then finding the correlated trajectories which develop for a representative range of initial positions of particle 2. When the initial position of particle 1, $z_1(0)$, is chosen to be equal to that of particle 2, $z_2(0)$, we obtain a bifurcation point. If $z_2(0) < z_1(0)$, then particle 2 has a negative velocity and $s_{z_2}$ decreases from 0 to $-\hbar/2$ whilst the corresponding particle 1 has a positive velocity and $s_{z_1}$ increases from 0 to $\hbar/2$. Analogous correlations are found if $z_2(0) > z_1(0)$. In figure 4 we illustrate the same phenomenon with a different choice of the constant $z_1(0)$.

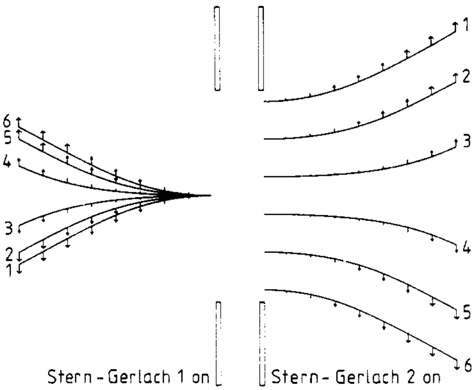


**Figure 3.** Correlated pairs of trajectories and spin vector orientations after the impulsive measurement of the spin in the $z'$ direction on both particles. $z_1(0)$ is constant, $z_2(0)$ is variable. The magnets may in principle be separated by any distance.

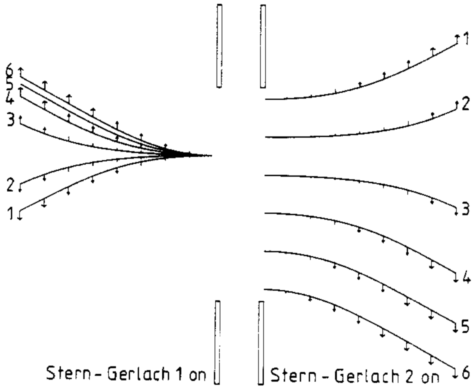


**Figure 4.** The situation of figure 3 with $z_1(0)$ equal to a different constant.

Returning to the general case ($\delta \neq 0$) it is evident from the above that regardless of whether one staggers the measurements in time or performs them simultaneously the statistical correlations will come out to be the same. The fate of a particular particle however will depend on when the measurements are carried out. Consider, for example, the correlated trajectories 3 in the figures. In figure 2 (§ 3.1) particle 1 will always move upward regardless of the initial position of particle 2. In figures 3 and 4 on the other hand (§ 3.2) particle 2 moves up or down depending on the value of $z_1(0)$. The identical statistics obtained in simultaneous or staggered measurements result in part therefore from quite different evolutions at the level of the individual particles which make up an ensemble. The non-local mechanism at work in the two cases is not the same, but they cannot be experimentally distinguished.

## 4. Resolution of the EPR 'paradox'

We have until now deliberately avoided referring to the EPR experiment as 'paradoxical' since no problems of logical consistency arise when the problem is formulated in the causal interpretation. The conventional reason why the features brought to light by EPR are felt to imply a paradox is as follows [8].

It is a straightforward prediction of the quantum formalism that if any component of the spin of particle 1 is measured and the result found to be $\pm(\hbar/2)$, then it can be immediately concluded that the same component of the spin of particle 2 (which is not measured) becomes definite and is equal to $\mp(\hbar/2)$. This will be true whatever component of particle 1 is measured since the singlet state is rotationally symmetric:

$$\frac{1}{\sqrt{2}}(u_+v_- - u_-v_+) = \frac{1}{\sqrt{2}}(u'_+v'_- - u'_-v'_+).$$

Unlike classical physics, it is claimed that only one component of the quantum mechanical spin of each particle can have a definite value at a given time—the two perpendicular components are indeterminate and are to be thought of as 'randomly fluctuating'. But we are free to measure the spin of particle 1 in any direction we choose, and so make definite the spin of particle 2 in any direction. Moreover, this choice may be made during the flight of the particles. The paradox then arises in the circumstance that particle 2 must somehow 'know' in which direction it is to be definite and in which it is to be fluctuating.

Einstein *et al* [6] used such an argument to show that quantum mechanics is incomplete; there should be an 'element of reality' corresponding to the spin of each particle since apparently without in any way disturbing one of the particles we can predict with certainty the value of its spin in any direction (having performed a measurement on the other particle). There is however no way in the usual formalism based on wavefunctions and operators to ascribe a simultaneous reality to spin components in perpendicular directions.

Bohr's answer [9] was that the form of the experiment and the content of the results form an unanalysable whole and that no paradox will arise if we refrain from drawing inferences from the results. The choice of different analysing field directions implies a series of mutually exclusive experimental arrangements whose details cannot be compared with one another.

The problem may be resolved in a different manner, however, in the causal interpretation which adopts Einstein's view that there is a reality independent of measurement and shows that this idea is completely consistent with the quantum mechanical formalism. As we have seen in the previous sections of this paper, the unjustified assumption in the conventional formulation of the 'paradox' outlined above is to suppose that if a reality is to be ascribed to entities such as 'spin' then this reality refers to the eigenvalues of operators, i.e. the dynamical variable is only realised and becomes definite (equal to an eigenvalue) on performing a measurement.

In our approach we suppose that both particles always have well defined trajectories and spin angular momenta components in all directions. The interaction of one of the particles with the Stern-Gerlach magnet implies the transformation of the wavefunction of the entire system, and consequently a change in the total spin-dependent quantum potential associated with the two bodies. As we have shown, when particle 1 interacts with the Stern-Gerlach device, it enters one or other of the emerging wavepackets while the quantum torque $\boldsymbol{T}_1$ rotates its spin vector so that it coincides with the eigenvalue of the spin operator being measured, and the quantum torque $\boldsymbol{T}_2$ rotates the spin of particle 2 to an equal and opposite value while the trajectory is unaffected.

The purpose of making simultaneous measurements on both particles is to show that these correlations in the particle's behaviour are brought about by an 'action at a distance' mechanism. Once the initial positions of the particles have been fixed, the outcome of the experiment is uniquely determined. The non-local spin-dependent quantum potential associated with the total quantum state rotates the spin vectors, as in the case of measurement on only one particle, and the total quantum potential ensures that the positions of the two particles are correlated in the required way. Changing the relative orientation of the magnets changes the total quantum state and hence implies different correlated motions. It is in this way that we incorporate the insights of Bohr concerning this process, but we also go on to explain how the correlations come about.

Although there is no classical interaction between the particles, they cannot be said to be non-interacting since they are tied by the quantum potential. Thus although we ascribe independent 'elements of reality' to individual quantum systems, the criterion for the existence of an independent 'element of reality' proposed by EPR [6] (if without in any way disturbing a system we can predict with probability unity the value of a dynamical quantity, then that quantity is an element of reality) is not applicable in the causal interpretation. The act of measurement performed on one particle does indeed bring about a disturbance in the properties of the other particle, via the quantum potential.

Note that in bringing about these correlations, the quantum potential and quantum torque do not transfer information. We do not know what result would have been obtained for particle 2 had we not made a measurement on particle 1. There is therefore no conflict between this form of non-locality and the relativistic requirement that no signal be transmitted faster than the speed of light. Nevertheless, Einstein–Podolsky–Rosen-type correlations in the properties of distantly separated systems may be felt to contradict at least the spirit of relativity which apparently requires actions only to be transmitted locally in order not to conflict with causality (i.e. that effects follow causes). A detailed treatment of this problem using the techniques of modern predictive mechanics shows, however, that no such contradiction arises [10]. Remarkably theories involving 'action at a distance' may be compatible with the principles of relativity if they satisfy certain constraints.

## 5. Bell's inequality

The preceding analysis enables us to see clearly the manner in which the assumptions made by Bell [7] in his derivation of an inequality that any local hidden variables theory must apparently satisfy are violated in the causal interpretation. In discussing the EPR spin experiment Bell supposed that the results of the two spin measurements are determined completely by a set of hidden variables $\lambda$ and made two crucial assumptions which he claimed should be satisfied by a local hidden variables theory.

(i) The result $A$ of measuring $\boldsymbol{\sigma}_1 \cdot \boldsymbol{a}$ on particle 1 is determined solely by $\boldsymbol{a}$ and $\lambda$, and the result $B$ of measuring $\boldsymbol{\sigma}_2 \cdot \boldsymbol{b}$ on particle 2 is determined solely by $\boldsymbol{b}$ and $\lambda$, where $\boldsymbol{a}$ and $\boldsymbol{b}$ are unit vectors with $\boldsymbol{a} \cdot \boldsymbol{b} = \cos \delta$. Thus

$$A = A(\boldsymbol{a}, \lambda) = \pm 1 \qquad B = B(\boldsymbol{b}, \lambda) = \pm 1.$$

Possibilities such as $A = A(\boldsymbol{a}, \boldsymbol{b}, \lambda)$, $B = B(\boldsymbol{a}, \boldsymbol{b}, \lambda)$ are excluded.

(ii) The normalised probability distribution of the hidden variables depends only on $\lambda$:

$$\rho = \rho(\lambda).$$

Possibilities such as $\rho = \rho(\lambda, \boldsymbol{a}, \boldsymbol{b})$ are excluded.

The first assumption was considered by Bell to be more important than the second in characterising the requirement of locality ((ii) was in fact only tacitly assumed).

From assumptions (i) and (ii) and the definition of the expectation value of the product of the results $A$ and $B$,

$$P(\boldsymbol{a}, \boldsymbol{b}) = \int d\lambda\, \rho(\lambda) A(\boldsymbol{a}, \lambda) B(\boldsymbol{b}, \lambda) \tag{5.1}$$

Bell deduced the inequality which is to be satisfied by the functions (5.1). This of course is violated by quantum mechanics; one cannot recover the result (3.2) from (5.1).

We now consider to what extent assumptions (i) and (ii) are valid in the causal interpretation. The hidden variables $\lambda$ are here the particle positions $\boldsymbol{x}_1, \boldsymbol{x}_2$ (the internal orientation spin vectors $\boldsymbol{s}_1, \boldsymbol{s}_2$ along the trajectories are determined by the positions and the wavefunction). In the case of staggered measurements, it follows from § 3.1 that $A = A(\boldsymbol{x}_1, \boldsymbol{a})$ and $B = B(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{a}\cdot\boldsymbol{b})$. When the measurements are performed simultaneously, it follows from (3.6) and (3.8) that which of the eventual results $\pm\hbar/2$ are obtained for $s_{z_1}$ and $s_{z_2}$ is determined solely by the initial positions of both particles and by $\boldsymbol{\delta}$, i.e. $A = A(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{a}\cdot\boldsymbol{b})$, $B = B(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{a}\cdot\boldsymbol{b})$. Thus assumption (i) is not valid in either case. Neither is assumption (ii) satisfied. In the causal interpretation the probability distribution of positions is derived from the quantum mechanical wavefunction which is a function of all the contributing parts of the process, including the orientations of the magnets (cf (3.3)). Therefore, in the causal interpretation, (5.1) should be modified to be

$$P(\boldsymbol{a}, \boldsymbol{b}) = \int d^3x_1\, d^3x_2\, \rho(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{a}\cdot\boldsymbol{b}) A(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{a}\cdot\boldsymbol{b}) B(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{a}\cdot\boldsymbol{b}) \tag{5.2}$$

from which one cannot deduce Bell's inequality. Using the probabilities (3.2*a*) in (5.2) we recover the quantum mechanical result (3.2)

$$\begin{aligned} P(\boldsymbol{a}, \boldsymbol{b}) &= \tfrac{1}{2}\sin^2\delta/2 + \tfrac{1}{2}\sin^2\delta/2 - \tfrac{1}{2}\cos^2\delta/2 - \tfrac{1}{2}\cos^2\delta/2 \\ &= -\cos\delta. \end{aligned}$$

Bell's inequality is therefore violated because the hidden variables are non-locally interconnected by the quantum potential derived from the total quantum state. It is in this sense that the causal interpretation implies non-local correlations in the properties of distantly separated systems.

## 6. Conclusion

We have shown here how the results of the EPR experiment can be accounted for in terms of a reality in which well defined and continuously variable quantities evolve in a deterministic manner according to the equations of motion of the causal interpretation. Our analysis illustrates that the fundamentally new feature of matter introduced by the quantum theory is wholeness, in which the behaviour of an individual particle is irreducibly connected with its context (expressed through the wavefunction). This arises most strikingly in the many-body case through non-local connections. The interactions regarded as measurements are those in which a particular variable of a 'measured' system (which already exists prior to the measurement) becomes correlated with a particular apparatus coordinate according to deterministic laws of evolution of the whole undivided system plus apparatus. In this sense the elements of reality of the quantum theory are essentially different to those of classical physics. Although we use the same terms (position, momentum, kinetic and potential energies) to describe a particle's motion, the individual and its relation with these attributes is not the same as in the classical domain.

Having established this feature of the quantum potential model, a further point to consider is the conditions under which correlations in the properties of distantly separated systems can be generally expected to occur. Non-locality arises when the wavefunction is not factorisable, but this is not a sufficient condition since under certain circumstances it may effectively factorise. For example, for a two-particle system in a harmonic oscillator potential obeying Bose-Einstein or Fermi-Dirac statistics, the particle motions are only correlated when the two wavepackets (each containing one particle) overlap appreciably [11]. When the particles are distantly separated (i.e. their packets do not overlap appreciably) they behave independently. A similar situation arises in our treatment of the EPR-Bohm case. The initial singlet state (3.1) describes two independently evolving particle motions. The quantum potentials $Q_i$, $i=1,2$, each depend on only one of the coordinates, and the spin-dependent additions (2.7) vanish. The non-local action of the quantum potential comes into existence only when a measurement is performed. It is probable that this is a feature specific to the idealised model discussed here. A more detailed theory of the spin correlation experiment is currently being developed which introduces internal angle degrees of freedom as independent variables in addition to the position coordinates. The theory based on the Pauli equation given above is recovered from this more general theory when one averages over the internal variables. Details will be published shortly.

Finally, it should be pointed out that although the recent experiments [12] testing the predictions of quantum mechanics in this case have been carried out on the polarisation states of photons, the principles involved in a causal treatment are the same as those outlined above.

**Acknowledgments**

One of us (PRH) would like to thank the Science and Engineering Research Council for a European fellowship which enabled this research to be carried out. The authors thank the referee for encouragement and useful comments.

**Appendix**

In order to obtain (3.3) we first derive the effect of the impulsive measurements by solving

$$i\hbar\frac{\partial\psi}{\partial t}=(W_1+W_2)\psi \tag{A1}$$

where (2.2) reduces to

$$W_1=\mu(H_0+z_1'H_0')\sigma_{z_1'} \qquad W_2=\mu(H_0+z_2H_0')\sigma_{z_2}$$

where $\sigma_{z'}=\sigma_z\cos\delta-\sigma_x\sin\delta$, $z'=-x\sin\delta+z\cos\delta$, $H_0$ is the uniform field and $H_0'$ is the field gradient in each Stern-Gerlach magnet. We ignore the effect of the fields in the $x$ and $x'$ directions which are necessarily present, as is easily seen from Maxwell's equations [13]. The initial wavefunction is (3.1). Since magnet 1 is not oriented in the $z$ direction we must expand the solution to (A1) in terms of all of the four basis functions:

$$\psi(z_1', z_2, t) = \sum_{a,b=\pm} \psi_{ab}(z_1', z_2, t) u_a v_b$$

where $\sigma_{z_1} u_\pm = \pm u_\pm$, $\sigma_{z_2} v_\pm = \pm v_\pm$. Substituting this expression in (A1) it is easy to find second-order differential equations satisfied by each $\psi_{ab}$. Expressing the solution in terms of the basis functions which correspond to the possible outcomes of the experiment, we find

$$\begin{aligned}\psi(z_1', z_2, T) = \frac{1}{\sqrt{2}} f_1(z_1') f_2(z_2) (\sin \tfrac{1}{2}\delta \exp\{-\mathrm{i}[2\Delta + (z_1' + z_2)\Delta']\} u_+' v_- \\ + \cos \tfrac{1}{2}\delta \exp[-\mathrm{i}(z_1' + z_2)\Delta'] u_+' v_- - \cos \tfrac{1}{2}\delta \exp[\mathrm{i}(z_1' - z_2)\Delta'] u_-' v_+ \\ + \sin \tfrac{1}{2}\delta \exp\{\mathrm{i}[2\Delta + (z_1' + z_2)\Delta']\} u_-' v_-) \end{aligned} \qquad \text{(A2)}$$

where $\sigma_{z_1'} u_\pm' = \pm u_\pm'$, $\Delta = \mu H_0 T/\hbar$, $\Delta' = \mu H_0' T/\hbar$ and $T$ is the time that each particle spends in the field.

The subsequent motion proceeds according to the free equation

$$\mathrm{i}\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial z_1'^2} + \frac{\partial^2}{\partial z_2^2}\right)\psi \qquad \text{(A3)}$$

with (A2) as initial wavefunction. To find the solution to (A3) we first Fourier analyse the initial packets $f_1, f_2$. The coefficient of $u_+' v_+$ in (A2), say, is then

$$\frac{1}{\sqrt{2}} \sin \tfrac{1}{2}\delta \, \mathrm{e}^{-2\mathrm{i}\Delta} \frac{1}{2\pi} \int g_1(k_1') g_2(k_2) \exp\{\mathrm{i}[(k_1' - \Delta')z_1' + (k_2 - \Delta')z_2]\} \, \mathrm{d}k_1' \, \mathrm{d}k_2 \qquad \text{(A4)}$$

where $g_1(k_1')$, $g_2(k_2)$ are normalised packets centred around $k_1' = 0$, $k_2 = 0$ respectively. As time passes, the $(k_1', k_2)$th Fourier component picks up a factor $\exp[-\mathrm{i}(\omega_{k_1'} + \omega_{k_2})t]$ where

$$\omega_{k_1'} = \frac{\hbar}{2m}(k_1' - \Delta')^2 \qquad \omega_{k_2} = \frac{\hbar}{2m}(k_2 - \Delta')^2.$$

(A4) thus becomes

$$\begin{aligned}\frac{1}{2\pi\sqrt{2}} \sin \tfrac{1}{2}\delta \, \mathrm{e}^{-2\mathrm{i}\Delta} \int g_1(k_1') g_2(k_2) \exp\{\mathrm{i}[(k_1' - \Delta')z_1' - (\hbar t/2m)(k_1' - \Delta')^2 \\ + (k_2 - \Delta')z_2 - (\hbar t/2m)(k_2 - \Delta')^2]\} \, \mathrm{d}k_1' \, \mathrm{d}k_2.\end{aligned}$$

The centre of the configuration space wavepacket occurs where the phase has an extremum, i.e. where

$$z_1' = -\hbar t \Delta'/m \qquad z_2 = -\hbar t \Delta'/m$$

so that the centres of the wavepackets emerging from the magnets move in the same direction (relative to the local analysing fields), as is to be expected from the $u_+' v_+$ part of the solution. The other three terms in (A3) develop similarly and describe the other possible combinations for the outcome of the experiment. Each of the particles enter one or the other packets at the exits to the magnets depending on their initial positions in the packets at the entrances to the magnets. It is assumed that the momenta imparted to the packets by the inhomogeneous fields are sufficiently great that the spreading of the packets will not mask the spin-dependent deflection and so a classical separation of the two beams emerging from each magnet is ensured.

The final states of each of the apparatuses (packet coordinates) are thus non-overlapping and those packets which the particles do not enter can be dropped from further attention. The measurement is complete when devices are placed beyond the magnets to detect which beams the particles actually entered. This irreversible stage of the process merely tells us what has already happened and there is no need to invoke the 'wavefunction collapse' hypothesis.

The final step in deriving (3.3) is to substitute explicit expressions for the packets $g_1, g_2$ in the four terms of the form (A4). Writing

$$g_1(k_1') = (\sigma\sqrt{\pi})^{-1/2}\exp(-k_1'^2/2\sigma^2) \qquad g_2(k_2) = (\sigma\sqrt{\pi})^{-1/2}\exp(-k_2^2/2\sigma^2)$$

performing the integrations and rearranging, we deduce (3.3).

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